Dynamic firm behaviour within an uncertain environment

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RESEARCH MEMORANDUM

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DYNAMIC FIRM BEHAVIOUR WITHIN AN UNCERTAIN ENVIRONMENT

Peter M. Kort

FEW 311

Dynamic Firm Behaviour within an Uncertain Environment

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Dynamic Firm Behaviour within an Uncertain Environment**)

In this paper we study the impact of an uncertain environment on the optimal dynamic investment policy of a value maximizing firm. Starting-point of the analysis is the model designed by Bensoussan and Lesourne [2]. To establish the influence of uncertainty on the shareholders' time preference rate we connect an adapted version of this model with the Intertemporal Capital Asset Pricing Model (see [11]). Among the most interesting results obtained in this way is a new formula for the shareholders' time preference rate. In this formula we combine the investment and cash decision. The cash can absorb the uncertainty, so the discount rate is equal to the riskless market intrest rate if we are dealing with a firm in a strong liquidity position.

I. Introduction

The dynamic theory of the firm has been a fruitful area for interesting scientific contributions. Recent books and papers include [6], [7], [9] and [12]. These studies, however, all have in common that the models under consideration are deterministic. The purpose of this paper is to extend these contributions by adding another dimension: uncertainty.

The first stochastic dynamic model of the firm was designed by Bensoussan and Lesourne [2]. A main difference between this model and the deterministic models mentioned before is the presence of cash. Within an uncertain

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**) The proofs of the propositions stated in the article can be obtained from the author upon request.

environment earnings can fall below the expense level, so net cash out flows may occur. Therefore a certain amount of cash is needed to meet the firm's obligations during such periods. In deterministic models there is no such reason for holding cash and therefore we can conclude that we need stochastic models of the firm to analyse the firm's cash decision. Another difference with the deterministic models is, that now the planning horizon is endogeneously determined, namely by the point of time that the amount of cash becomes negative, i.e. when the firm goes bankrupt.

One of the results of the static theory of the Capital Asset Pricing Model is that the discount rate depends on the amount of risk the firm has to deal with (see [5], p. 195). Like in deterministic models of the firm, Bensoussan and Lesourne assume the shareholders' time preference rate to be constant. But due to the uncertain environment, the firm has to deal with risk in this model. Therefore it seems interesting to incorporate a dynamic version of the CAPM in the stochastic dynamic model of the firm.

In section 2 we present our dynamic model of the firm in which the Bensoussan and Lesourne model is extended by changing the objective from dividend maximization into the maximization of the utility stream of dividends. Now, we can apply the CAPM approach, which is only valid under the assumption of risk-averse investors. In section 3 the model is solved while in section 4 the Intertemporal Capital Asset Pricing Model (see [11]) is incorporated. In section 5 we summarize our findings.

II. Model formulation

In this section the stochastic dynamic model of the firm is presented. The stochastic aspect of the model is incorporated in the earnings function, which can be expressed as:

$$R(K) = S(K)\ (1 + \sigma V) \tag{1}$$

in which:

K = capital good stock.

$R(K)$ = earnings function.

$S(K)$ = usual deterministic earnings function (see e.g. [9]), $\frac{dS}{dK} > 0$, $S(0) = 0$, $\left.\frac{dS}{dK}\right|_{K=0} > i$, where i = the shareholders' time preference rate.

V = Gaussian stochastic variable, $E(V) = 0$, $Var(V) = 1$, $E(V(T), V(\hat{T})) = 0$ if $T \neq \hat{T}$.

σ = a constant.

To apply the technique of dynamic programming we have to rewrite (1) into an Ito stochastic differential equation (see e.g. [1]). Before we do this, first notice that $V(T)dT$ can be formally expressed as $dB(T)$, where $B(T)$ is a standard Wiener process (see [13], p. 296). If we multiply (1) by dT we obtain:

$$R(K)dT = S(K)dT + \sigma S(K)dB \tag{2}$$

We now formulate the rodel in symbols; afterwards the interpretation of the model will be given:

$$\underset{\dot{K},D}{\text{maximize:}}\ E\left[\int_0^Z V(D)e^{-iT}\,dT\right] \tag{3}$$

s.t.

$$dK = \dot{K}\, dT \tag{4}$$

$$dM = (S(K)-\dot{K}-D)dT + \sigma S(K)dB \tag{5}$$

$$D \geq 0,\ \dot{K} \geq 0,\ S(K) - \dot{K} - D \geq 0 \qquad (6), (7), (8)$$

$$K(0) = K_0 > 0,\ M(0) = M_0 > 0 \qquad (9), (10)$$

in which:

D = dividend

M = cash

T = time

$V(D)$ = utility function of the shareholders, $\frac{dV}{dD} > 0$, $\frac{d^2V}{dD^2} < 0$,

$V(0) = 0$

Z = planning horizon

i = shareholders' time preference rate.

The firm behaves as if it maximizes the shareholders' value of the firm. This value is expressed as the mathematical expectation of the discounted utilitystream of dividends where the utility function is concave. The firm is bankrupt as soon as M becomes negative. In this way the planning horizon Z is endogeneously determined by being the first instant reached for which $M < 0$ (3). We further suppose that there are no depreciations and that investments are irreversible ((4), (7)). Due to (2) and (5), we derive that the firm can spend its earnings in three directions: increase cash, invest the money and pay dividend. Dividend policy is bounded by a rational lower bound (6) and further we assume that at any time the firm does not spend more money on investments and dividends, than the mathematical expectation of the earnings (8). Debt is not included in the model, because we want to focus first on the cash management problem. Bensoussan and Lesourne [3] have carried out some numerical experiments on a stochastic model that includes the possibility for the firm of borrowing.

If we should change the objective from maximizing a utilitystream of dividends into dividend maximization:

$$\int_0^Z De^{-iT} dT \tag{11}$$

we obtain the model designed by Bensoussan and Lesourne [2]. In [8] this model is extensively treated and the optimal solution is improved. The most realistic part of this solution is presented in figure 1.

[Place Figure 1 here]

Figure 1 shows that depending on the level of cash and capital goods the firm carries out one of the following policies:

$\dot{M}$-policy: The firm keeps its cash if the amount of equipment is high enough while the cash-situation is poor.

$\dot{K}$-policy: The firm invests if the amount of equipment is low, while there is plenty of cash to limit the risk of bankruptcy.

D-policy: The firm distributes dividends if M and K are such that the marginal profitability of investment is too small to justify additional growth and the amount of cash available high enough to guarantee a sufficiently sure situation.

In the remainder of this paper we shall use the objective described by (3).

III. Solution

First, we introduce:

$$U(M(t),\ K(t)) = \max_{\substack{\dot{K}, D \geq 0 \\ \dot{K}+D \leq S(K)}} E\left[\int_t^Z V(D)e^{-iT}dT\right] \tag{12}$$

U is equal to the maximization of the mathematical expectation of the discounted utilitystream of dividends and can be interpreted as the value of the firm. Notice that U depends completely on M and K and not on the value of t, because the planning horizon is state dependent and not a fixed point of time.

Assuming that the partial derivatives $\frac{\partial U}{\partial M}$, $\frac{\partial U}{\partial K}$ and $\frac{\partial^2 U}{\partial M^2}$ exist, the following proposition can be stated:

Proposition 1

The Hamilton-Jacobi-Bellman equation of the problem under consideration can be expressed by:

$$iU = \max_{\substack{\dot{K},D\geq 0 \\ \dot{K}+D\leq S(K)}} \left\{V(D) + \frac{\partial U}{\partial M}(S(K) - \dot{K} - D) + \frac{\partial U}{\partial K}\dot{K}\right\} + \frac{1}{2}\sigma^2 S^2(K)\frac{\partial^2 U}{\partial M^2} \tag{13}$$

To the Hamilton-Jacobi-Bellman equation we adjoin the boundary equation:

$$U(0,K) = 0 \tag{14}$$

Depending on the relative size of $\frac{\partial U}{\partial M}$, $\frac{\partial U}{\partial K}$ and $\frac{dV}{dD}$ for different values of D, the policies maximizing the righthand side of (13) differ. The analysis shows that five such policies have to be considered, which can be easily economically interpreted since:

$\frac{\partial U}{\partial K}$ = the marginal increase of the value of the firm due to an additional investment of one dollar.

$\frac{\partial U}{\partial M}$ = the marginal increase of the value of the firm due to one extra dollar kept in cash.

$\frac{dV}{dD}$ = the marginal increase of the value of the firm due to an additional dollar used to distribute dividends.

Notice that $\frac{dV}{dD} = \frac{\partial U}{\partial D}$, because the money paid out as dividend will leave the firm immediately and therefore it has no long-term effects. The five optimal policies are the following:

<u>Investment Policy</u>: $dM = \sigma S(K)dB$, $D = 0$, $dK = S(K)dT$

optimal if:

$$\frac{\partial U}{\partial K} \geq \max_{0 \leq D \leq S(K)} \left[\frac{dV}{dD}, \frac{\partial U}{\partial M}\right] \tag{15}$$

Thus for this policy it is marginally more interesting:

- to invest than to pay out dividend
- to invest than to increase cash.

<u>Cash Policy</u>: $dM = S(K)dT + \sigma S(K)dB$, $D = 0$, $dK = 0$

optimal if:

$$\frac{\partial U}{\partial M} \geq \max_{0 \leq D \leq S(K)} \left[\frac{dV}{dD}, \frac{\partial U}{\partial K}\right] \tag{16}$$

Due to (16) we can conclude that for this policy it is marginally more interesting:

- to increase cash than to pay out dividend
- to increase cash than to invest.

<u>Dividend Policy</u>: $dM = \sigma S(K)dB$, $D = S(K)$, $dK = 0$

optimal if:

$$\left.\frac{dV}{dD}\right|_{D=S(K)} \geq \max \left[\frac{\partial U}{\partial K}, \frac{\partial U}{\partial M}\right] \tag{17}$$

For this policy it is marginally more interesting:

- to pay out dividend than to invest
- to pay out dividend than to increase cash.

Cash/Dividend Policy: $dM = (S(K)-D)dT + \sigma S(K)dB,\ D \geq 0,\ dK = 0)$

optimal if:

$$\frac{dV}{dD} = \frac{\partial U}{\partial M} \geq \frac{\partial U}{\partial K} \qquad (18)$$

Due to (18) and the concavity of V(D) it is marginally more interesting:

- to use a part of the expected earnings for paying out dividend and the other part to increase cash than to invest
- to use a part of the expected earnings for paying out dividend and the other part to increase cash than to use all expected earnings to increase cash
- to use a part of the expected earnings for paying out dividend and the other part to increase cash than to use all expected earnings for paying out dividend.

Investment/Dividend Policy: $dM = \sigma S(K)dB,\ D \geq 0,\ dK = (S(K)-D)dT$

optimal if:

$$\frac{dV}{dD} = \frac{\partial U}{\partial K} \geq \frac{\partial U}{\partial M} \qquad (19)$$

From (19) and the concavity of V(D) we derive that it is marginally more interesting:

- to use a part of the expected earnings for paying out dividend and the other part to invest than to increase cash

- to use a part of the expected earnings for paying out dividend and the other part to invest than to use all expected earnings for investment
- to use a part of the expected earnings for paying out dividend and the other part to invest than to use all expected earnings for paying out dividend.

After we have established the five policies that can be optimal, we now derive at what level of M and K which of these policies will be carried out. To do so, we will devide the M-K plane in five different regions, each of them corresponding to one of the five optimal policies. In this way we get the following regions: investment-region, cash-region, dividend-region, cash/dividend-region and investment/dividend-region.
In contrast with the model of Bensoussan and Lesourne [2] that maximizes dividend (see figure 1), in the present model the boundary between the cash-region and the dividend-region does not exist for K positive. This is, because in the cash-region it holds that $\frac{\partial U}{\partial M} \geq \left.\frac{dV}{dD}\right|_{D=0}$ and in the dividend-region $\frac{\partial U}{\partial M}$ must be less or equal to $\left.\frac{dV}{dD}\right|_{D=S(K)}$. Due to the concavity of V(D) we can conclude that for K positive $\left.\frac{dV}{dD}\right|_{D=0} > \left.\frac{dV}{dD}\right|_{D=S(K)}$, so in the M-K plane the cash-region and the dividend-region have no points in common. Therefore, there will always be a cash/dividend-region between the cash- and the dividend-region. The same reasoning can be applied to argue that the investment/dividend-region must always be situated between the investment-region and the dividend-region.
The following Propositions show where boundaries are situated if M goes to infinity.

Proposition 2

On the boundary between the cash-region and the cash/dividend-region and on the boundary between the cash/dividend-region and the dividend-region it holds that also K must have an infinite value if M goes to infinity.

Proposition 3

If M goes to infinity, the boundaries between the investment/dividend-region and the dividend-region and between the investment-region and the investment/dividend-region are both situated on the level K*, for which $\left.\frac{dS}{dK}\right|_{K=K^*} = i$.

About the (non) existence of the intersection points of the boundaries we can establish the following rules:

- For K positive the boundaries between cash and cash/dividend $\left[\frac{\partial U}{\partial M} = \left.\frac{dV}{dD}\right|_{D=0}\right]$ and between cash/dividend and dividend $\left[\frac{\partial U}{\partial M} = \left.\frac{dV}{dD}\right|_{D=S(K)}\right]$ do not intersect, because on one point $\frac{\partial U}{\partial M}$ cannot have two different values.
- Following the same reasoning for K positive, we can demonstrate that the boundaries between investment and investment/dividend $\left[\frac{\partial U}{\partial K} = \left.\frac{dV}{dD}\right|_{D=0}\right]$ and between investment/dividend and dividend $\left[\frac{\partial U}{\partial K} = \left.\frac{dV}{dD}\right|_{D=S(K)}\right]$ do not intersect.
- For K positive, the boundaries between cash and cash/dividend $\left[\frac{\partial U}{\partial M} = \left.\frac{dV}{dD}\right|_{D=0}\right]$ and between investment/dividend and dividend $\left[\frac{\partial U}{\partial K} = \left.\frac{dV}{dD}\right|_{D=S(K)}\right]$ do not intersect, because it is never optimal to invest in the direct neighbourhood of the intersection-point.

- Following the same reasoning we can argue that the boundaries between cash/dividend and dividend $\left[\frac{\partial U}{\partial M} = \frac{dV}{dD}\Big|_{D=S(K)}\right]$ and between investment and investment/dividend $\left[\frac{\partial U}{\partial K} = \frac{dV}{dD}\Big|_{D=0}\right]$ do not intersect.

Due to the complexity of the model under consideration we were not able to derive an almost complete solution as can be done for the model with dividend maximization. But if we start with some reasonable assumptions we can enable the optimal policies for the firm depending on the different levels of M and K. For these assumptions we use the economic interpretation of figure 1, thus with other words we take the solution of the model with dividend maximization as a starting-point for deriving the optimal solution of this model. Briefly stated, the assumptions are:

a) The firm distributes dividends if M and K are available high enough.
b) The firm keeps its cash if K is high enough while the cash situation is poor.
c) The firm invests if cash is such that the risk of bankruptcy is limited, while the amount of equipment is low.

By using the above derived properties of the boundaries we are now able to construct the optimal solution, which is presented in figure 2.

[Place Figure 2 here]

Like in the model of Bensoussan and Lesourne (see figure 1), in this model it is not optimal to invest if K is greater than K*. The reason is that, due to the concavity of S(K) the expected marginal earnings $\left(= \frac{dS}{dK}\right)$ then fall below the return the shareholders demand (=i). This feature also plays an important role in the solutions of deterministic models (see e.g. [9]).

In comparison with the solution represented by figure 1, the present solution contains two more regions in which a mixed cash dividend- ($\dot{M}$/D) and a mixed investment dividend-policy ($\dot{K}$/D) will be carried out, respectively.

Concerning the cash/dividend-region, on the boundary with the cash-region ($\dot{M}$) it holds that D = 0 and on the boundary with the dividend-region (D) D is equal to S(K). In between D has such a value that $\frac{\partial U}{\partial M} = \frac{dV}{dD}$, so the increase of the value of the firm due to one unit extra cash is equal to the marginal utility of dividend.

Concerning the investment/dividend-region, on the boundary with the investment-region ($\dot{K}$) it holds that $\dot{K}$ = S(K) and D = 0 and on the boundary with the dividend-region $\dot{K}$ is equal to zero and D is equal to S(K). In the rest of this region $\dot{K}$ and D behave such that $\frac{\partial U}{\partial K} = \frac{dV}{dD}$.

If we drop assumption b, the solutions presented in figure 3 arise.

[Place Figure 3 here]

In figure 3a, the shareholders do not want the firm to increase the amount of cash, even if cash is almost zero. This will be optimal in a very risky environment. Because of the bankruptcy risk the shareholders want to obtain dividend as soon as possible. They do not want to raise cash first, because there is a risk of the firm going bankrupt before the dividend payout starts. Of course, this solution will only be optimal in very extreme situations such as under severe threats of war, revolution, etc. Concerning figure 3 we are able to prove the following Proposition:

Proposition 4

A necessary condition for figure 3a to be optimal is that for all K it must hold that:

$$\left.\frac{V(D)}{\frac{dV}{dD}.D}\right|_{D=S(K)} \leq \frac{\sigma\sqrt{i}}{\sqrt{2}} \tag{20}$$

Concerning figure 3b, the cash/dividend-region covers the K-axis for those K which satisfy the following expression:

$$\left.\frac{V(D)}{\frac{dV}{dD}.D}\right|_{D=S(K)} > \frac{\sigma\sqrt{i}}{\sqrt{2}} \tag{21}$$

The result of Proposition 4 can be nicely interpreted from economical point of view. First, notice that $\frac{V(D)}{\frac{dV}{dD}.D}$ is a measure of the concavity of the utility function. Relatively spoken shareholder with a concave utility function does not want large amounts of dividends and therefore he likes a mixed cash/dividend policy. A shareholder with a large time preference rate wants to obtain a large amount of dividends as soon as possible and if investment is very risky, shareholders want to obtain dividends immediately because of the high risk of bankruptcy. If (20) holds, investment is that risky (σ) and/or the shareholders' time preference rate (i) is that large, that the influence of the concavity of the utility function is not big enough to provide for the optimality of the mixed cash/dividend policy. If the amount of cash is low, the optimality of this policy is guaranteed for those K that satisfy relation (21).

IV. The model extended with the Intertemporal Capital Asset Pricing Model

In this section we connect the Intertemporal Capital Asset Pricing Model with our dynamic model of the firm. The ICAPM was invented by Merton [11] and is based on the following assumptions, which are briefly stated here (see also [4], p. 604):

- perfect capital market
- prices of securities are lognormally distributed
- investors have homogeneous expectations
- investors are risk-averse.

If we assume that these assumptions are satisfied, the following ICAPM-relation becomes applicable:

$$\alpha_F - r = \frac{\lambda\sigma_{Fm}}{\sigma_m} \tag{22}$$

in which:

α_F = the firm's expected rate of return per unit time.

r = riskless borrowing - lending rate.

σ_m = standard deviation of the rate of return per unit time of the market portfolio.

σ_{Fm} = covariance between the rate of return per unit time of the firm and the market portfolio.

$\lambda = \dfrac{\alpha_m - r}{\sigma_m}$ = market price per unit risk, where

α_m = expected rate of return per unit time of the market portfolio.

Following a method described by Constantinides [4], we derive α_F and σ_{Fm} for our model. First, we apply Ito's lemma (see [10], p. 89) to the value function:

$$dU = U(M+dM,\ K+dK) - U(M,K) = \left[\frac{\partial U}{\partial M}\left[S(K) - \dot{K} - D\right] + \frac{\partial U}{\partial K}\dot{K} + \right.$$

$$\left. + \frac{\sigma^2}{2} S^2(K) \frac{\partial^2 U}{\partial M^2}\right] dT + \frac{\partial U}{\partial M}\sigma S(K) dB \tag{23}$$

After assuming that the firm is optimally controlled and using (23), we can state the firm's rate of return:

$$\frac{U(M+dM,\ K+dK) + V(D)dT - U(M,K)}{U(M,K)} = \frac{1}{U(M,K)}\left[V(D) + \frac{\partial U}{\partial M}(S(K) - \dot{K} - D) + \right.$$

$$\left. + \frac{\partial U}{\partial K}\dot{K} + \frac{\sigma^2}{2} S^2(K) \frac{\partial^2 U}{\partial M^2}\right] dT + \frac{\frac{\partial U}{\partial M}\sigma S(K)}{U(M,K)} dB \tag{24}$$

From (13) and (24) we derive:

$$\alpha_F = i \tag{25}$$

$$\sigma_{Fm} = \rho_{Fm}\sigma_m\sigma S(K)\,\frac{\frac{\partial U}{\partial M}}{U} \tag{26}$$

in which:

ρ_{Fm} = instantaneous correlation coëfficient between the firm's return and the market return.

After substituting (25) and (26) in (22) we get the following expression for the time preference of the shareholder:

$$i = r + \lambda\rho_{Fm}\sigma S(K)\,\frac{\frac{\partial U}{\partial M}}{U} \tag{27}$$

So, analogous to one of the results of the static CAPM, the shareholders' time preference rate consists of the sum of the riskless market intrest rate and a risk premium. The risk premium depends on the market price per unit risk (=λ), the correlation coëfficient between the returns of the firm and the market (= ρ_{Fm}), the standard deviation of the earnings function (= $\sigma S(K)$) and the sensitivity of the value of the firm with regard to a marginal change in the amount of cash $\left[= \frac{\partial U}{\partial M}/U\right]$. This makes sense, because the amount of risk of the return of the firm's investment depends on cash. If the amount of cash is low there is a high risk of bankruptcy, thus now the influence of a marginal change in cash on the value of the firm is high. If the firm has a large amount of cash, then one unit increase or decrease of this amount does not influence the expected utility stream of dividends, thus then a marginal change in cash has no influence on the value of the firm. So, we can conclude that the risk premium will be high if the amount of cash is low.

We now state the following Proposition:

Proposition 5

If M goes to infinity the boundaries between $\dot{K}/D$ and D and between $\dot{K}$ and $\dot{K}/D$ approach an assymptote which is situated on the level $\hat{K}$, for which $\left.\frac{dS}{dK}\right|_{K=\hat{K}} = r$.

In the dynamic model of the firm in which the CAPM is not incorporated, the assymptote corresponds to that level of K, for which $\frac{dS}{dK} = i$. The reason for the difference between this result and Proposition 5 is that now the shareholders' time preference rate is equal to r if M has an infinite value. This is, because there is no risk of bankruptcy, so $\frac{\partial U}{\partial M}$ is equal to zero and from (27) we derive that the shareholders' time preference is then equal to the riskless market intrest rate. To conclude: if M is large, the risk has disappeared out of the model.

V. Summary

In this paper the analysis of deterministic dynamic models of the firm is extended by adding a stochastic component in the earnings function. Due to this extension earnings may fall below the expense level and therefore the firm needs cash to meet its obligations during those periods. Starting-point is the pathbreaking work of Bensoussan and Lesourne [2] who analysed a stochastic model with dividend maximization. Using the technique of dynamic programming they proved that - depending on the amount of capital goods, the amount of cash, the shareholders' time preference rate and the variance of the earnings - it is optimal for the firm to choose one of the following three ways of spending its expected earnings: increase the amount of cash, invest the money or pay it out as dividend.

We extend the Bensoussan and Lesourne model by changing the objective from dividend maximization into maximizing a utilitystream of dividends where the utilityfunction is concave and by incorporating a dynamic version of the Capital Asset Pricing Model invented by Merton [11]. Using a method described by Constantinides [4], we were able to derive a new formula for the shareholders' time preference rate, which consists of the riskless intrest rate and a risk premium. We further demonstrate that, in contrast with the Bensoussan and Lesourne model, also a mixed investment dividend-policy and a mixed cash dividend-policy could be optimal for the firm to carry out. Another interesting result is that the shareholders' time preference rate is equal to the riskless market intrest rate if the firm deals with a large amount of cash, thus having no bankruptcy risk.

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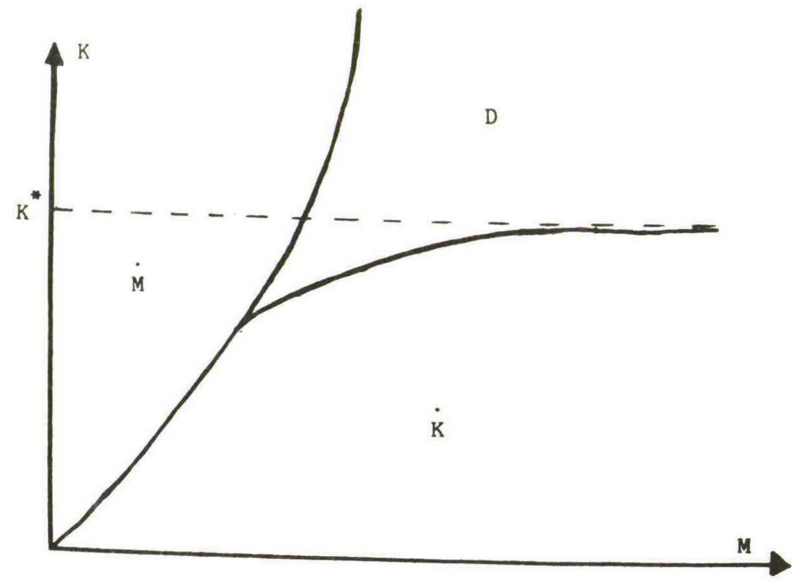
Figure captions

figure 1. The most realistic part of the optimal solution of Bensoussan and Lesourne's model.

figure 2. The optimal solution of the model under the assumptions a, b and c.

figure 3 figure 3a figure 3b.

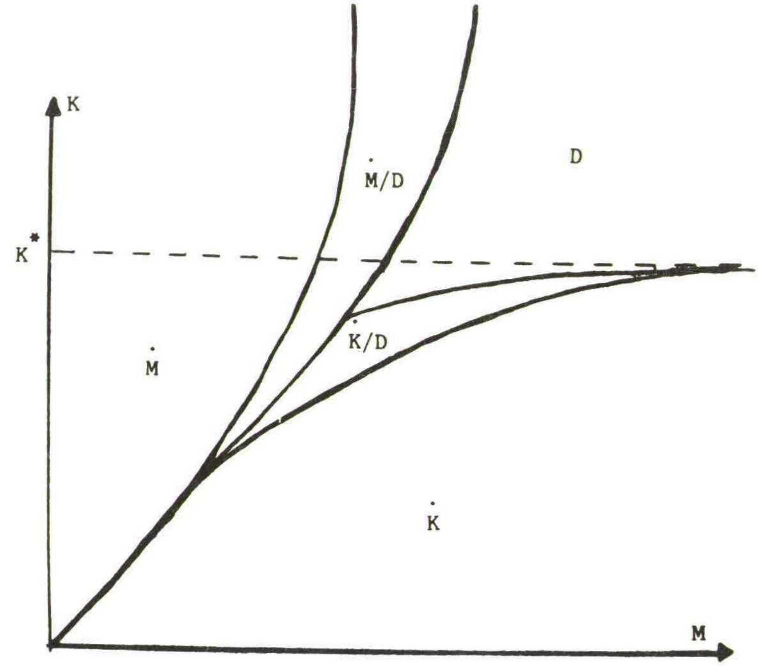
The optimal solution of the model under the assumptions a and c.



in which:

$$\left.\frac{dS}{dK}\right|_{K=K^*} = i$$

Figure 1



in which:

$$\left.\frac{dS}{dK}\right|_{K=K^*} = i$$

Figure 2

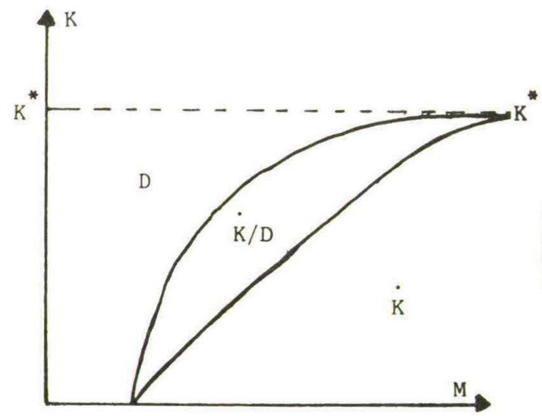
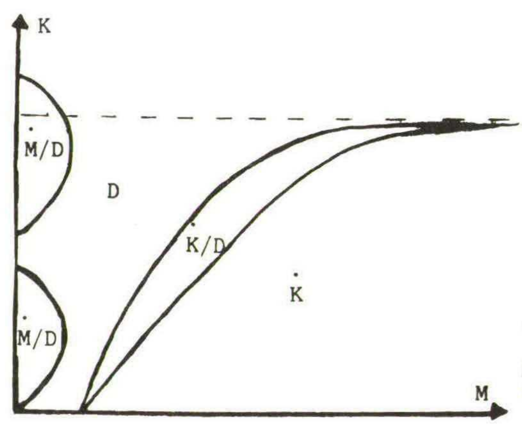


Figure 3a



in which:

$$\left.\frac{dS}{dK}\right|_{K=K^*} = i$$

Figure 3b

IN 1987 REEDS VERSCHENEN

242 Gerard van den Berg
Nonstationarity in job search theory

243 Annie Cuyt, Brigitte Verdonk
Block-tridiagonal linear systems and branched continued fractions

244 J.C. de Vos, W. Vervaat
Local Times of Bernoulli Walk

245 Arie Kapteyn, Peter Kooreman, Rob Willemse
Some methodological issues in the implementation
of subjective poverty definitions

246 J.P.C. Kleijnen, J. Kriens, M.C.H.M. Lafleur, J.H.F. Pardoel
Sampling for Quality Inspection and Correction: AOQL Performance Criteria

247 D.B.J. Schouten
Algemene theorie van de internationale conjuncturele en strukturele afhankelijkheden

248 F.C. Bussemaker, W.H. Haemers, J.J. Seidel, E. Spence
On (v,k,λ) graphs and designs with trivial automorphism group

249 Peter M. Kort
The Influence of a Stochastic Environment on the Firm's Optimal Dynamic Investment Policy

250 R.H.J.M. Gradus
Preliminary version
The reaction of the firm on governmental policy: a game-theoretical approach

251 J.G. de Gooijer, R.M.J. Heuts
Higher order moments of bilinear time series processes with symmetrically distributed errors

252 P.H. Stevers, P.A.M. Versteijne
Evaluatie van marketing-activiteiten

253 H.P.A. Mulders, A.J. van Reeken
DATAAL - een hulpmiddel voor onderhoud van gegevensverzamelingen

254 P. Kooreman, A. Kapteyn
On the identifiability of household production functions with joint products: A comment

255 B. van Riel
Was er een profit-squeeze in de Nederlandse industrie?

256 R.P. Gilles
Economies with coalitional structures and core-like equilibrium concepts

257 P.H.M. Ruys, G. van der Laan
Computation of an industrial equilibrium

258 W.H. Haemers, A.E. Brouwer
Association schemes

259 G.J.M. van den Boom
Some modifications and applications of Rubinstein's perfect equilibrium model of bargaining

260 A.W.A. Boot, A.V. Thakor, G.F. Udell
Competition, Risk Neutrality and Loan Commitments

261 A.W.A. Boot, A.V. Thakor, G.F. Udell
Collateral and Borrower Risk

262 A. Kapteyn, I. Woittiez
Preference Interdependence and Habit Formation in Family Labor Supply

263 B. Bettonvil
A formal description of discrete event dynamic systems including perturbation analysis

264 Sylvester C.W. Eijffinger
A monthly model for the monetary policy in the Netherlands

265 F. van der Ploeg, A.J. de Zeeuw
Conflict over arms accumulation in market and command economies

266 F. van der Ploeg, A.J. de Zeeuw
Perfect equilibrium in a model of competitive arms accumulation

267 Aart de Zeeuw
Inflation and reputation: comment

268 A.J. de Zeeuw, F. van der Ploeg
Difference games and policy evaluation: a conceptual framework

269 Frederick van der Ploeg
Rationing in open economy and dynamic macroeconomics: a survey

270 G. van der Laan and A.J.J. Talman
Computing economic equilibria by variable dimension algorithms: state of the art

271 C.A.J.M. Dirven and A.J.J. Talman
A simplicial algorithm for finding equilibria in economies with linear production technologies

272 Th.E. Nijman and F.C. Palm
Consistent estimation of regression models with incompletely observed exogenous variables

273 Th.E. Nijman and F.C. Palm
Predictive accuracy gain from disaggregate sampling in arima - models

274 Raymond H.J.M. Gradus
The net present value of governmental policy: a possible way to find the Stackelberg solutions

275 Jack P.C. Kleijnen
A DSS for production planning: a case study including simulation and optimization

276 A.M.H. Gerards
A short proof of Tutte's characterization of totally unimodular matrices

277 Th. van de Klundert and F. van der Ploeg
Wage rigidity and capital mobility in an optimizing model of a small open economy

278 Peter M. Kort
The net present value in dynamic models of the firm

279 Th. van de Klundert
A Macroeconomic Two-Country Model with Price-Discriminating Monopolists

280 Arnoud Boot and Anjan V. Thakor
Dynamic equilibrium in a competitive credit market: intertemporal contracting as insurance against rationing

281 Arnoud Boot and Anjan V. Thakor
Appendix: "Dynamic equilibrium in a competitive credit market: intertemporal contracting as insurance against rationing

282 Arnoud Boot, Anjan V. Thakor and Gregory F. Udell
Credible commitments, contract enforcement problems and banks: intermediation as credibility assurance

283 Eduard Ponds
Wage bargaining and business cycles a Goodwin-Nash model

284 Prof.Dr. hab. Stefan Mynarski
The mechanism of restoring equilibrium and stability in polish market

285 P. Meulendijks
An exercise in welfare economics (II)

286 S. Jørgensen, P.M. Kort, G.J.C.Th. van Schijndel
Optimal investment, financing and dividends: a Stackelberg differential game

287 E. Nijssen, W. Reijnders
Privatisering en commercialisering; een oriëntatie ten aanzien van verzelfstandiging

288 C.B. Mulder
Inefficiency of automatically linking unemployment benefits to private sector wage rates

289 M.H.C. Paardekooper
A Quadratically convergent parallel Jacobi process for almost diagonal matrices with distinct eigenvalues

290 Pieter H.M. Ruys
Industries with private and public enterprises

291 J.J.A. Moors & J.C. van Houwelingen
Estimation of linear models with inequality restrictions

292 Arthur van Soest, Peter Kooreman
Vakantiebestemming en -bestedingen

293 Rob Alessie, Raymond Gradus, Bertrand Melenberg
The problem of not observing small expenditures in a consumer expenditure survey

294 F. Boekema, L. Oerlemans, A.J. Hendriks
Kansrijkheid en economische potentie: Top-down en bottom-up analyses

295 Rob Alessie, Bertrand Melenberg, Guglielmo Weber
Consumption, Leisure and Earnings-Related Liquidity Constraints: A Note

296 Arthur van Soest, Peter Kooreman
Estimation of the indirect translog demand system with binding non-negativity constraints

IN 1988 REEDS VERSCHENEN

297 Bert Bettonvil
Factor screening by sequential bifurcation

298 Robert P. Gilles
On perfect competition in an economy with a coalitional structure

299 Willem Selen, Ruud M. Heuts
Capacitated Lot-Size Production Planning in Process Industry

300 J. Kriens, J.Th. van Lieshout
Notes on the Markowitz portfolio selection method

301 Bert Bettonvil, Jack P.C. Kleijnen
Measurement scales and resolution IV designs: a note

302 Theo Nijman, Marno Verbeek
Estimation of time dependent parameters in lineair models using cross sections, panels or both

303 Raymond H.J.M. Gradus
A differential game between government and firms: a non-cooperative approach

304 Leo W.G. Strijbosch, Ronald J.M.M. Does
Comparison of bias-reducing methods for estimating the parameter in dilution series

305 Drs. W.J. Reijnders, Drs. W.F. Verstappen
Strategische bespiegelingen betreffende het Nederlandse kwaliteitsconcept

306 J.P.C. Kleijnen, J. Kriens, H. Timmermans and H. Van den Wildenberg
Regression sampling in statistical auditing

307 Isolde Woittiez, Arie Kapteyn
A Model of Job Choice, Labour Supply and Wages

308 Jack P.C. Kleijnen
Simulation and optimization in production planning: A case study

309 Robert P. Gilles and Pieter H.M. Ruys
Relational constraints in coalition formation

310 Drs. H. Leo Theuns
Determinanten van de vraag naar vakantiereizen: een verkenning van materiële en immateriële factoren